Testing Nested Models

- Two models are *nested* if both contain the same terms and one has at least one additional term.

- Example:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x2 + \beta_3 x_1 x_2 + \epsilon \tag{1}$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x2 + \beta_3 x_1 x_2 + \beta_4 x_1^2 + \beta_5 x_2^2 + \epsilon \tag{2}$$

- Model (1) is *nested within* model (2).

- Model (1) is the **reduced** model and model (2) is the **full** model.

Testing Nested Models (cont'd)

- How do we decide whether the more complex (full) model contributes additional information about the association between y and the predictors?

- In example above, this is equivalent to testing $H_0 : \beta_4 = \beta_5 = 0$ versus H_a : at least one $\beta \neq 0$.

- Test consists in comparing the SSE for the reduced model (SSE_R) and the SSE for the complete model (SSE_C).

- $SSE_R > SSE_C$ always so question is whether the drop in SSE from fitting the complete model is 'large enough'.

Testing Nested Models (cont'd)

- We use an $F-$test to compare nested models, one with k parameters (reduced) and another one with $k+p$ parameters (complete or full).

- Hypotheses: $H_0 : \beta_{k+1} = \beta_{k+2} = ... = \beta_{k+p} = 0$ versus H_a : At least one $\beta \neq 0$.

- Test statistic: $F = \frac{(SSE_R - SSE_C)/ \text{ \# of additional } \beta' s}{SSE_C/[n-(k+p+1)]}$

- At level α, we compare the $F-$statistic to an F_{ν_1,ν_2} from table, where $\nu_1 = p$ and $\nu_2 = n-(k+p+1)$.

- If $F \geq F_{\alpha,\nu_1,\nu_2}$, reject H_0.

Testing Nested Models (cont'd)

- See Example 4.10 on page 233.

- Steps are:

  1. Fit complete model with $k + p$ β's and get SSE_C.
  2. Fit reduced model with k β's and get SSE_R.
  3. Set up hypotheses and choose α value.
  4. Compute $F-$statistic and compare to table F_{α,ν_1,ν_2}.

- If test leads to rejecting H_0, then at least one of the additional terms in the model contributes information about the response.

Testing Nested Models (cont'd)

- Parsimonious models are preferable to big models as long as both have similar predictive power.

- A parsimonious model is one with a small number of predictors.

- If models are not nested, cannot use the $F-$test above to choose between one and another. Must rely on other sample statistics such as R_a^2 and $RMSE$.

- In the end, choice of model is subjective.